0.1 stepped pressure equilibrium code : fc00aa

1. Given vector position returns force.

2. The force vector, $\mathbf{F}(\boldsymbol{\xi})$, is a combination of the pressure-imbalance Fourier harmonics, $([[p+B^2/2]]\sqrt{g}^i)_{m,n}$, where $i \equiv$`Iusesg`. and the spectral condensation constraints arising from minimization of $M = \sum_j (m_j^p + n_j^q)(R_j^2 + Z_j^2)/2$ with respect to tangential variations, $\delta R \equiv R_\theta \delta u$ and $\delta Z \equiv Z_\theta \delta u$.

3. The vector, $\boldsymbol{\xi}$, represents the geometrical degrees of freedom of the internal interfaces. This vector is 'unpacked' and the `Rbc`, `Zbs` arrays are assigned.

4. The routine `ex00aa` is called to extrapolate innermost surface / magnetic axis.

5. The routine `ih00aa` is called to interpolate the coordinate harmonics and construct the global coordinates.

6. The following routines are called in parallel:

   (a) `ma00ab` : allocates sub-grid geometric arrays, `igss`, etc. in each annulus;

   (b) `ma00aa` : calls `me00ab` on sub-sub-grid to compute matrix elements on sub-grid; calculates `igss` etc.;

   (c) `cb02aa` : computes Fourier harmonics of $p + B^2/2$ and spectral contraints on interfaces;

   (d) `vo00aa` : calculate volume integral;

   (e) `fu00aa` : calculate volume integrals of pressure, B^2 and $\mathbf{A}\cdot\mathbf{B}$.

   (f) `ma00ab` : deallocates `igss`, etc. ;

7. The routine `bc00aa` broadcasts data that needs to be shared. and the force vector is constructed.

0.1.1 theory

1. The energy functional, plus the angle spectral constraint, in each annulus is

$$F_l = \int_{\mathcal{V}_l} dv \left(\frac{p}{\gamma-1} + \frac{B^2}{2}\right) - \frac{\mu_l}{2}\int_{\mathcal{V}_l} dv\,(\mathbf{A}\cdot\mathbf{B}) - \nu_l \int_{\mathcal{V}_l} dv\, p^{1/\gamma} + \lambda_l \frac{1}{2}\sum_j (m_j^p + n_j^q)(R_j^2 + Z_j^2). \tag{1}$$

   The Lagrange multipliers, μ_l and ν_l are to be determined below. The factor λ_l is to be decreased until the minimization of the spectral constraint (a purely numerical constraint) does not impact on force-balance (a physical constraint).

2. The variation in F_l due to variations in the pressure is

$$\delta F_l = \int_{\mathcal{V}_l} dv\, \delta p \left(\frac{1}{\gamma-1} - \frac{\nu_l p^{1/\gamma}}{\gamma p}\right) \tag{2}$$

   Hereafter, we assume that $\nu_l p^{1/\gamma} = \gamma p/(\gamma-1)$, so that the pressure is constant in each volume.

3. The Euler-Lagrange equations for the variations in the field and interface geometry are connected: if the interface geometry changes, the magnetic field must also change in order to ensure that the field remain tangential to the interfaces. On the interfaces we use $\delta\mathbf{A} = \delta\boldsymbol{\xi}\times\mathbf{B}$ and derive

$$\delta F_l = \int_{\mathcal{V}_l} dv\, \delta\mathbf{A}\cdot(\nabla\times\mathbf{B} - \mu_l\mathbf{B}) - \int_{\partial\mathcal{V}_l} \delta\boldsymbol{\xi}\cdot d\mathbf{S}\, \left(p + B^2/2\right) + \lambda_l \sum_j (m_j^p + n_j^q)(R_j\delta R_j + Z_j \delta Z_j) \tag{3}$$

   Hereafter, we assume that $\nabla\times\mathbf{B} = \mu_l\mathbf{B}$, and that μ_l is *initially* adjusted to satisfy the interface transform constraint.

4. Position is $\mathbf{x} = R\,\hat{r} + Z\,\hat{z}$, where $\hat{r} = \cos\phi\,\mathbf{i} + \sin\phi\,\mathbf{j}$ and $\hat{\phi} = -\sin\phi\mathbf{i} + \cos\phi\,\mathbf{j}$. The coordinate transformation is $R = R(s,\theta,\zeta)$, $\phi = -\zeta$, $Z = Z(s,\theta,\zeta)$.

5. The area element is $d\mathbf{S} \equiv \mathbf{e}_\theta \times \mathbf{e}_\zeta d\theta d\zeta = [RZ_\theta \hat{r} + (Z_\theta R_\zeta - R_\theta Z_\zeta)\hat{\phi} - RR_\theta \hat{z}]d\theta d\zeta$

6. The variation in position is $\delta\boldsymbol{\xi} = \delta R\,\hat{r} + \delta Z\,\hat{z}$, and so $\delta\boldsymbol{\xi}\cdot d\mathbf{S} = R(\delta R Z_\theta - \delta Z R_\theta)d\theta d\zeta$.

fc00aa.h last modified on 2011-10-21 ;